JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 9, Number 4, Fall 1997

GALERKIN APPROXIMATION WITH QUADRATURE FOR THE SCREEN PROBLEM IN $\mathbf{R}^3$

R.D. GRIGORIEFF AND I.H. SLOAN

ABSTRACT. We study a Galerkin method with quadrature for the single-layer equation on a two-dimensional plate Γ which has the form of a union of rectangles. The trial space consists of piecewise constant functions on a partition of Γ into rectangles, which is assumed to be quasi-uniform. A semi-discrete scheme is obtained by approximating the $L^2(\Gamma)$ inner product in the definition of the Galerkin matrix elements by composite quadrature rules. More precisely, the integral over each rectangular element is replaced by a composite quadrature rule, obtained by subdividing the rectangle into M^2 congruent subrectangles of the same shape as the original, and applying a scaled version of a basic quadrature rule to each subrectangle. The basic quadrature rule is required to have only interior nodes; in this way possible singularities which can be present on the boundary of the rectangles of the partition are not encountered. High precision of the quadrature rules is not necessary. The stability of the semi-discrete scheme is proved, under appropriate conditions, if the subdivision of each rectangle of the partition is fine enough; more precisely, if $M \geq M_0$, with M_0 independent of the partition when the basic quadrature rule is exact for polynomials of degree 1. Error estimates are derived which show that the semi-discrete Galerkin approximations will converge at the same rate as the corresponding Galerkin approximations in some norms.

1. Introduction. This paper is concerned with a semi-discrete Galerkin method for solving the single-layer equation

$$Vu = f \tag{1}$$

for a plane plate. More specifically, the equation is

$$Vu(x) := \frac{1}{4\pi} \int_\Gamma \frac{u(y)}{|x-y|}\, dy, \quad x \in \Gamma, \tag{2}$$

where $\Gamma \subset \mathbf{R}^2$ is a bounded region which is the union of a finite number of rectangles, with all the rectangle edges either parallel or

Received by the editors on January 22, 1997, and in revised form on July 24, 1997.