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RESOLVENT ESTIMATES FOR ABEL INTEGRAL OPERATORS AND THE REGULARIZATION OF ASSOCIATED FIRST KIND INTEGRAL EQUATIONS

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ABSTRACT. In this paper resolvent estimates for Abel integral operators are provided. These estimates are applied to deduce regularizing properties of Lavrentiev's m-times iterated method as well as iterative schemes (with the discrepancy principle as corresponding parameter choice or stopping rule, respectively) for solving the corresponding Abel integral equations of the first kind.

1. Introduction.

1.1. *Introductory remarks.* Various applications lead to Abel integral equations of the first kind $Au = f_*$, where

$$(1.1) \qquad (Au)(\xi) = \frac{\beta^{1-\alpha}}{\Gamma(\alpha)} \int_\xi^a \frac{\eta^{\beta-1} u(\eta)}{(\eta^\beta - \xi^\beta)^{1-\alpha}}\, d\eta, \quad \xi \in [0,a],$$

or

$$(1.2) \qquad (Au)(\xi) = \frac{\beta^{1-\alpha}}{\Gamma(\alpha)} \int_0^\xi \frac{\eta^{\beta-1} u(\eta)}{(\xi^\beta - \eta^\beta)^{1-\alpha}}\, d\eta, \quad \xi \in [0,a],$$

(with $0 < \alpha < 1$, $0 < a < \infty$, $0 < \beta$, and with Γ denoting Euler's gamma function), see Subsection 1.2 for one of these applications.

In this paper we provide resolvent estimates for Abel integral operators (1.1) and (1.2) (operating from X into X for the spaces $X = L^p([0,a], \xi^{\beta-1}\, d\xi)$, $p \in [1,\infty]$, and $X = C[0,a]$, respectively), i.e., we provide norm estimates of $(\lambda I + A)^{-1}$ for specific $\lambda \in \mathbf{C}$, with I denoting the identity operator in the underlying space X.

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