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A STEFAN/MULLINS-SEKERKA TYPE PROBLEM WITH MEMORY

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ABSTRACT. Existence is proven for the system

(0.1)

$$a_1 u_t + a_2 w_t = \int_{-\infty}^{t} k(t-s)\Delta u(s)\, ds \quad (x,t) \in \Omega \times (0,T)$$

$$a_3 w_t = \Delta \mu \quad (x,t) \in \Omega \times (0,T)$$

$$\mu + 2u \in \partial \Gamma(w) \quad (x,t) \in \Omega \times (0,T)$$

where

$$\Gamma(w) := \begin{cases} \int_\Omega |\nabla w| < \infty & |w| \leq 1 \text{ a.e.} \\ \infty & \text{otherwise} \end{cases}$$

for arbitrary $T > 0$ on a smooth bounded domain $\Omega \subset \mathbf{R}^n$, $n = 1, 2$, or 3 via the inclusion of a relaxation dynamic, for initial data $(u, w) \in L^2(\Omega) \times BV(\Omega)$ and for "prehistory" $u_h \in L^2(\mathbf{R}^-; H^2(\Omega))$. Here u denotes the temperature field, w a conserved phase variable, and μ the chemical potential. Neumann boundary conditions are assumed for μ and the heat flux or normal derivative of the temperature field is prescribed. The kernel k is assumed to be of positive type. The system (0.1) represents a coupled Stefan/Mullins-Sekerka type problem which has recently been derived by formal asymptotics from a formulation of the conserved phase-field equations which allows for memory effects in the temperature field, [**7**].

1. Introduction. In this paper we study existence and uniqueness for the following Stefan/Mullins-Sekerka problem in which memory

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