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LOCALIZATION AND POST PROCESSING FOR THE GALERKIN BOUNDARY ELEMENT METHOD APPLIED TO THREE-DIMENSIONAL SCREEN PROBLEMS

ERNST P. STEPHAN AND THANH TRAN

ABSTRACT. We study local error estimates for various Galerkin schemes (Galerkin schemes with quasi-uniform or graded meshes, and the augmented Galerkin method) applied to weakly singular and hypersingular integral equations on open surfaces in $\mathbf{R}^3$. The results are given for a large scale of Sobolev norms, even in some norms that are not defined globally. In the case of the weakly singular integral equation where the highest orders of convergence achieved are in negative Sobolev norms, we establish from the Galerkin solutions new solutions that converge in the L^2-norm to the exact solution in these orders.

1. Introduction. The solutions of elliptic boundary value problems in $\mathbf{R}^3\backslash\Gamma$, where Γ is an open surface in $\mathbf{R}^3$, have special singular forms at the boundary γ of Γ, regardless of whether γ is a smooth or polygonal curve. When those problems are reformulated, via the direct method, into boundary integral equations, the solutions of the latter inherit those singularities. These singularities affect the rate of global convergence of numerical schemes, e.g., the Galerkin boundary element methods. To recover the high order of convergence associated with smooth and closed surfaces, either augmented boundary elements or mesh grading is necessary. However, if the given data are sufficiently smooth, the solutions to the integral equations are smooth locally, i.e., away from the singularities. Then there arises the following question. Is the accuracy of the approximation better in regions of smooth behavior of the exact solutions? Another problem is faced when we want to observe the highest order of global convergence when it is achieved in a negative norm. Is there any effective post-processing method to obtain that highest order for the local convergence in the L^2-norm, which can

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