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ON THE SOLUTION OF NONLINEAR VOLTERRA CONVOLUTION EQUATION WITH POWER NONLINEARITY

NIKOLAI K. KARAPETYANTS, ANATOLY A. KILBAS AND MEGUMI SAIGO

ABSTRACT. The Volterra nonlinear convolution integral equation

$$\varphi^m(x) = a(x)\int_0^x k(x-t)\varphi(t)\,dt + f(x)$$
$$0 < x < d \leq \infty$$

with $m > 0$, $m \neq 1$, and real functions $a(x)$, $k(u)$ and $f(x)$ is studied. Local estimates and asymptotic properties near zero for its solution $\varphi(x)$ are given, provided that $a(x)$, $k(u)$ and $f(x)$ have power asymptotic behaviors near zero. The integral equation with a power kernel being solvable in closed form is considered.

1. Introduction. The Volterra nonlinear convolution integral equation

$$(1) \qquad \varphi^m(x) = a(x)\int_0^x k(x-t)\varphi(t)\,dt + f(x), \qquad 0 < x < d \leq \infty$$

with $m \in \mathbf{R}$ arises in nonlinear theory of wave propagation [**11**] and water perlocation [**8, 18**]. Such an equation with $m > 0$, $m \neq 1$, was studied in [**1, 4, 6, 9, 16, 17, 18**] for $a(x) = 1$ and in [**2, 3, 5, 7**] for the general case. It is proved that the solvability of equation (1) is different in the cases $m > 1$ and $0 < m < 1$. In the former case the corresponding homogeneous equation, $f(x) \equiv 0$, can have a nontrivial solution, which was first proved in [**21**], and the papers above were devoted to investigate problems concerning the existence and uniqueness for the solution $\varphi(x)$ of the nonhomogeneous equation (1), the stability of such a solution and the method of successive approximation to construct this solution. Some results about the uniqueness of the solution of equation

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