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ON THE SOLUTION OF A SPECIAL KIND OF SINGULAR INTEGRAL EQUATION

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ABSTRACT. An explicit inversion formula for Cauchy singular integral equations is used to numerically approximate the solution. We deal with the case where the solution is not continuously differentiable but is in a special class of Hölder continuous functions. We get results for the order of convergence in the supremum norm and for a weighted square norm.

1. Introduction. Let L be a simple, closed smooth curve in $\mathbf{C}$ and $\psi \in C^\alpha(L, \mathbf{C})$, $\alpha \in (0,1)$. Here $C^\alpha(L,\mathbf{C})$ is the space of Hölder continuous functions on L with the usual norm $\|\cdot\|_{C^\alpha(L,\mathbf{C})}$. The solution of the following singular integral equation

$$\frac{1}{\pi i}\int_L \frac{\varphi(t)}{t-t_0}\,dt = \psi(t_0), \qquad t_0 \in L, \tag{1}$$

is given by

$$\varphi(t_0) = \frac{1}{\pi i}\int_L \frac{\psi(t)}{t-t_0}\,dt, \qquad t_0 \in L, \tag{2}$$

and we have $\varphi \in C^\alpha([0,2\pi], \mathbf{C})$ [**6**, Section 27, formula (A)]. This means that we only need a suitable quadrature formula to find an approximation for φ.

A typical example for the application of the above formula (2) is the following:

Given a function $\psi \in C^\alpha([0,2\pi],\mathbf{R})$, periodic, with

$$\psi(0) = \psi(2\pi) \tag{3}$$

and

$$\int_0^{2\pi} \psi(x)\,dx = 0, \tag{4}$$

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