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ON THE INTEGRAL EQUATION $f(x)-(c/L(x))\int_0^{L(x)} f(y)\,dy=g(x)$ WHERE $L(x)=\min\{ax,1\}$, $a>1$.

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ABSTRACT. In the present paper we consider a function f which is a solution of the integral equation

$$f(x) - \frac{c}{L(x)} \int_0^{L(x)} f(y)\,dy = g(x).$$

Here g is a given, smooth function defined on the interval $[0,1]$, $c \in (0,1)$ is a constant, and L is a continuous piecewise linear function through the points $(0,0)$, $(a^{-1},1)$, (1,1), where also $a > 1$ is a constant. We mainly focus our attention on the regularity properties of f. Away from the origin the regularity is analyzed by applying the Banach fixed point theorem, while near the origin we get a singular expansion for f by using the Mellin transform techniques.

1. Introduction. The aim of the present work is to study the integral equation

$$f(x) - \frac{c}{L(x)} \int_0^{L(x)} f(y)\,dy = g(x) \in C[0,1], \tag{1.1a}$$

where

$$L(x) = \begin{cases} ax & \text{if } 0 \le x \le a^{-1}, \\ 1 & \text{if } a^{-1} \le x \le 1, \end{cases} \tag{1.1b}$$

and a and c are constants such that

$$a > 1, \quad 0 < c < 1.$$

The work is motivated by certain problems of linear elasticity theory. As observed recently, an integral equation very similar to (1.1) arises

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