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EFFICIENT ALGORITHMS FOR THE p-VERSION OF THE BOUNDARY ELEMENT METHOD

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ABSTRACT. We investigate the p-version of the boundary element Galerkin method for a first kind integral equation. We present a-priori driven algorithms which yield sparse Galerkin matrices and do not destroy the convergence properties of the boundary element method. Further, we show that the additive Schwarz method is nearly an optimal preconditioner for the Galerkin systems. Numerical results confirm the efficiency of our methods.

1. Introduction. The p-version of the boundary element method (BEM) is known to be very efficient in view of its convergence properties. For problems with singularities it converges twice as fast as the usual h-version, see, e.g., [**13**]. To exploit these advantages in practice one has also to take care of an efficient implementation. One aspect is the treatment of generally fully occupied system matrices which are characteristic of the BEM. Another aspect is the fast solution of the linear systems which is of course not peculiar to the BEM. In case of the h-version these aspects have been investigated by several authors, see, e.g., [**1, 8, 7, 20, 18**]. In contrast, the structures of the system matrices of the p-version are not known to be under investigation so far. Also the construction of optimal preconditioners for the p-version has just started to be under investigation, see [**15**].

For simplicity we will concentrate on the weakly singular integral equation

$$V\Phi(x) = g(x), \qquad x \in \Gamma \tag{1}$$

where V is the single layer operator defined as

$$V\Phi(x) := -\frac{1}{\pi} \int_\Gamma \Phi(y) \log |x - y| \, ds_y.$$

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