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IMPROVED CONVERGENCE RATES FOR SOME DISCRETE GALERKIN METHODS

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ABSTRACT. We show how to improve the estimate of the convergence rate of a number of discrete polynomially-based Galerkin methods for Fredholm and Cauchy singular integral equations. This has been accomplished by sharpening the bounds on the quadrature errors in a manner analogous to that of Joe [**14**] for spline-based methods. These results are then extended to establish the convergence of some discrete Galerkin methods for one-dimensional hypersingular equations and some boundary integral equations on the sphere in $\mathbf{R}^3$.

Introduction. In a number of recent papers we have examined the convergence rate of various polynomially-based Galerkin methods for Fredholm and singular integral equations [**8–12**]. The convergence analysis took into account the effects of quadrature errors and for Fredholm equations may be seen as complementary to similar results of Atkinson and Bogomolny [**4**], Joe [**14**] and Spence and Thomas [**20**] using spline bases. In the case of splines, the above authors were able to obtain optimal convergence rates, i.e., convergence rates equal to that of the best approximation to the solution by splines of a given order. For polynomial approximations we were unable to do this, in part because of over estimation of various quadrature errors. In this paper, making use of an argument analogous to that of Joe [**14**] for spline approximations, we are able to improve our estimate of the convergence rate from $0(n^{-r+1})$ to $0(n^{-r+\frac{1}{2}})$ where n is the degree of the polynomial approximation. This seems the best that can be done by perturbation techniques.

The paper is divided into five sections. In Section 2 we review our previous results for Fredholm and Cauchy singular equations and indicate where improvements to our prior analysis can be made. In Section 3 we provide new estimates of quadrature errors generalizing those in [**10–12**]. These are then applied to improve the convergence rates

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