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NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS OF BARBASHIN TYPE: TOPOLOGICAL AND MONOTONICITY METHODS

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ABSTRACT. The purpose of this paper is to illustrate the applicability of topological and monotonicity methods to the solution of nonlinear integro-differential equations of Barbashin type. Such equations arise in the mathematical modelling of various transport phenomena. We show first how to solve initial value problems for nonlinear Barbashin equations by means of a classical fixed point theorem due to M.A. Krasnosel'skij. Afterwards, we apply a nonclassical fixed point principle for nonlinear operators in so-called K-normed spaces to a certain boundary value problem for Barbashin equations. The main step consists here in transforming the boundary value problem into an equivalent operator equation involving Uryson-type integral operators. Finally, we show how to use Minty's monotonicity principle to prove (unique) solvability of a Barbashin equation containing Hammerstein-type integral operators.

1. A fixed point theorem by Krasnosel'skij.

In 1955, M.A. Krasnosel'skij proved the following fixed point principle:

Theorem 1 [**9**]. *Let E be a Banach space, $G_1 : E \to E$ a contraction, and $G_2 : E \to E$ a continuous compact operator. Suppose that there exists a nonempty convex closed bounded set $M \subset E$ such that $G_1(M) + G_2(M) \subseteq M$. Then the operator $G_1 + G_2$ has a fixed point in M.*

Obviously, Theorem 1 bridges the "gap" between the classical fixed point principles of Banach-Caccioppoli and Schauder. Theorem 1 is

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