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ON THE INVERSION OF HIGHER ORDER WIENER-HOPF OPERATORS

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ABSTRACT. It is known that the Banach algebra generated by classical Wiener-Hopf operators on the half-line is an algebra with symbol. This concept yields, in particular, a Fredholm criterion and an index formula. In the present paper we introduce a different symbol for the finitely generated algebra. It is based on matricially coupling of operators and implies a representation of a generalized inverse in terms of matrix factorization. Some examples demonstrate how to use these results for a discussion of properties of the solution of singular equations.

1. Introduction. Our main objective is to construct generalized inverses for particular classes of operators which are somehow related to singular operators. This desire comes from mathematical physics where analytical formulas are needed to obtain direct information about the qualitative behavior of the solution of a linear operator equation, for instance asymptotic expansion, which cannot be obtained from the knowledge of a Fredholm pseudoinverse.

More precisely, if L^- denotes a generalized inverse of a bounded linear operator $L \in \mathcal{L}(X, Y)$ acting between Banach spaces, i.e. if

$$LL^-L = L \tag{1.1}$$

holds, then the equation

$$Lf = g \tag{1.2}$$

(for given $g \in Y$ and unknown $f \in X$) is solvable if and only if $LL^-g = g$ and the general solution in this case reads explicitly

$$f = L^-g + (I - L^-L)h, \qquad h \in X. \tag{1.3}$$

One of the most popular examples where L^- can be represented in closed analytical form is the Wiener-Hopf equation on the half-line

$$Wf(x) = \lambda f(x) + \int_0^\infty k(x-y)f(y)\,dy = g(x) \tag{1.4}$$