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QUADRATURES FOR BOUNDARY INTEGRAL EQUATIONS OF THE FIRST KIND WITH LOGARITHMIC KERNELS

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ABSTRACT. We consider boundary integral equations of the first kind with logarithmic kernels on smooth closed or open contours in R^2. Instead of solving the first kind equations directly, we propose a fully discrete quadrature method for the equivalent second kind equations with kernels defined by Cauchy singular integrals simply using the trapezoidal integration rules. Convergence of the method is completely analyzed. It is proved that the order of convergence is $O(1/n^{2k})$, where n is the number of nodes in the quadrature formula and $2k+2$ is the degree of smoothness of the righthand side function of the equation. Numerical examples are presented to confirm the theoretical estimate.

1. Introduction. Recently there has been considerable interest in numerical solutions of boundary integral equations of the first kind with logarithmic kernels (see [**3, 4, 7, 8, 10, 11**] and references cited therein). These equations arise from reformulations of Dirichlet problems for Laplace's equation in the plane, using single-layer potentials. The classical integral equation methods for boundary value problems usually reduce the boundary value problems into integral equations of the second kind. This is because the Fredholm theory and collective compact operator theory provide simple approaches for both theoretical analysis and numerical analysis for second kind equations. However, in the last decade, engineers and mathematicians have realized that first kind boundary integral equation reformulations using the single-layer potentials allow simple numerical solutions of practical problems since in many applications the density function of the single-layer potential of a boundary value problem is the final target of computation.

In this paper we study a Nyström method for integral equations of the first kind with a logarithmic kernel

$$\int_S g(Q)\log|P-Q|\,dS(Q) = h(P), \qquad P\in S, \tag{1.1}$$

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