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GRID-VALUED CONDITIONAL YEH-WIENER INTEGRALS AND A KAC-FEYNMAN WIENER INTEGRAL EQUATION

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ABSTRACT. In this paper we establish several results involving grid-valued conditional Yeh-Wiener integrals of the type

$$E(F(x)|x(s_1,\cdot),\dots,x(s_m,\cdot),x(*,t_1),\dots,x(*,t_n)).$$

We develop a formula for converting these grid-valued conditional Yeh-Wiener integrals into ordinary Yeh-Wiener integrals. We also obtain a Cameron-Martin translation theorem for these integrals. More importantly, we evaluate these conditional expectations for functionals F of the form

$$F(x)=\exp\bigg\{\int_0^T\int_0^S \phi(u,v,x(u,v))\,du\,dv\bigg\}$$

by solving a Kac-Feynman type Wiener integral equation.

1. Introduction. For $Q=[0,S]\times[0,T]$ let $C(Q)$ denote Yeh-Wiener space, i.e., the space of all real-valued continuous functions $x(s,t)$ on Q such that $x(0,t)=x(s,0)=0$ for every (s,t) in Q. Yeh [**11**] defined a Gaussian measure m_y on $C(Q)$ (later modified in [**14**]) such that as a stochastic process $\{x(s,t),(s,t)\in Q\}$ has mean $E[x(s,t)]=\int_{C(Q)}x(s,t)m_y(dx)=0$ and covariance $E[x(s,t)x(u,v)]=\min\{s,u\}\min\{t,v\}$. Let $C_w\equiv C[0,T]$ denote the standard Wiener space on $[0,T]$ with Wiener measure m_w. Yeh [**13**] introduced the concept of the conditional Wiener integral of F given X, $E(F\mid X)$, and for the case $X(x)=x(T)$ obtained some very useful results including a Kac-Feynman integral equation.

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