JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 8, Number 1, Winter 1996

ON WAVE EQUATIONS WITH BOUNDARY DISSIPATION OF MEMORY TYPE

GEORG PROPST AND JAN PRÜSS

ABSTRACT. The undamped wave equation on an open domain of arbitrary dimension and boundary of class C^1 is considered. On parts of the boundary the normal derivative of the solution equals the convolution of its time derivative with a measure of positive type. This setting subsumes standard dissipative boundary conditions as well as the interaction with viscoelastic boundary materials. Applying methods for evolutionary integral equations to a variational formulation of the problem, existence, uniqueness and regularity of the solution to the wave equation is proven under minimal regularity assumptions on the initial conditions and forcing functions. To evaluate the versatility of a parametrized model, least-squares fits to physical data are presented.

1. Introduction. A basic linear model for the evolution of sound in a compressible fluid is the system of partial differential equations

$$\begin{aligned} &\rho v_t(t,x) + \operatorname{grad} p(t,x) = 0, \\ &\kappa p_t(t,x) + \operatorname{div} v(t,x) = 0, \qquad t>0, \quad x \in \mathbf{R}^n, \end{aligned} \tag{1.1}$$

where p denotes acoustic pressure and v the velocity field; cf., e.g., Leis [**6**]. In the sequel the equilibrium density ρ and the compressibility κ will be assumed to be constant and then w.o.l.g. to be equal to 1. Eliminating v from this system one obtains a wave equation for the pressure p.

$$p_{tt}(t,x) = \Delta p(t,x), \qquad t>0, \quad x \in \mathbf{R}^n. \tag{1.2}$$

When the fluid is enclosed in a region $\Omega \subset \mathbf{R}^n$, (1.2) has to be supplemented by conditions at $\partial\Omega$, the boundary of Ω. The energy conserving Dirichlet, Neumann and Robin conditions aside, the following

Accepted for publication on November 3, 1995.
AMS *Mathematics Classifications.* 35L05, 35L20, 45N05.
Key words and phrases. Wave equation, boundary conditions of memory type, dissipation, well-posedness, evolutionary integral equations.