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ON SUPERCONVERGENCE RESULTS AND NEGATIVE NORM ESTIMATES FOR PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. The purpose of this paper is to show how known negative norm estimates and superconvergence results applied to parabolic equations can be carried over to integro-differential equations of parabolic type. A quasi projection technique introduced earlier by Douglas, Dupont and Wheeler is modified to establish negative norm estimates in several space variables. Further, in a single space variable, knot superconvergence is also established. Finally, interior superconvergence estimates are also derived.

1. Introduction. In this paper, we discuss some superconvergence results and negative norm estimates for the following parabolic integro-differential equation

$$
\begin{aligned}
u_t + A(t)u &= \int_0^t B(t,\tau)u(\tau)\,d\tau + f \quad \text{in } \Omega \times J,\\
u &= 0 \quad \text{on } \partial\Omega \times J,\\
u(\cdot,0) &= u_0 \quad \text{in } \Omega\ .
\end{aligned}
\tag{1.1}
$$

Here, $u = u(x,t)$ and $f = f(x,t)$ are real valued functions in $\Omega \times J$, where Ω is a bounded domain in R^d with smooth boundary $\partial\Omega$, $J = (0,T]$, $T < \infty$ and $u_t = \partial u/\partial t$. Further, $A(t)$ is a selfadjoint, uniformly positive definite second order elliptic partial differential operator of the form

$$A(t) = -\sum_{i,j=1}^{d} \frac{\partial}{\partial x_j}\left(a_{ij}(x,t)\frac{\partial}{\partial x_i}\right) + a_0(x,t)I,$$

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