JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 8, Number 1, Winter 1996

STABILITY PROBLEMS OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH ABSTRACT VOLTERRA OPERATOR

YIZENG LI

Introduction. In this paper we study stability problems for the functional differential equations involving abstract Volterra operators under second kind initial value

$$\begin{cases} \dot{x}(t) = (Vx)(t), & t > t_0, \\ x(t) = \phi(t), & t \in [0, t_0), \\ x(t_0) = x^0 \in \mathbf{R}^n, \end{cases} \tag{1.1}$$

where V is a continuous Volterra operator acting on $L^2_{\mathrm{loc}}([0,\infty), \mathbf{R}^n)$, with $(V\theta)(t) \equiv \theta \in \mathbf{R}^n$, and $\phi \in L^2([0,t_0), \mathbf{R}^n)$, where θ is used to denote both the zero function and the zero vector throughout the article.

We first give the definitions of stability for the trivial solution (or zero solution, or equilibrium) of the system (1.1). Although there are many kinds of stability to be discussed, among them we emphasize five main stability concepts. They are *stability, uniform stability, asymptotic stability, uniformly asymptotic stability*, and *exponentially asymptotic stability.*

Then we shall present the necessary and sufficient conditions for the stabilities with regard to the trivial solution $x = \theta \in \mathbf{R}^n$ of the linear system

$$\begin{cases} \dot{x}(t) = (Lx)(t), & t > t_0, \\ x(t) = \phi(t), & t \in [0, t_0), \\ x(t_0) = x^0 \in \mathbf{R}^n, \end{cases} \tag{1.2}$$

where L is a linear continuous Volterra operator acting on $L^2_{\mathrm{loc}}([0,\infty), \mathbf{R}^n)$ with $(L\theta)(t) \equiv \theta \in \mathbf{R}^n$, and $\phi \in L^2([0,t_0), \mathbf{R}^n)$. These are new contributions.

Received by the editors on January 29, 1995, and in revised form on July 10, 1995.