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CONVERGENCE ESTIMATES FOR SOLUTION OF INTEGRAL EQUATIONS WITH GMRES

S.L. CAMPBELL, I.C.F. IPSEN, C.T. KELLEY, C.D. MEYER AND Z.Q. XUE

ABSTRACT. In this paper we derive convergence estimates for the iterative solution of nonsymmetric linear systems by GMRES. We work in the context of strongly convergent-collectively compact sequences of approximations to linear compact fixed point problems. Our estimates are intended to explain the observations that the performance of GMRES is independent of the discretization if the resolution of the discretization is sufficiently good. Our bounds are independent of the righthand side of the equation, reflect the r-superlinear convergence of GMRES in the infinite dimensional setting, and also allow for more than one implementation of the discrete scalar product. Our results are motivated by quadrature rule approximation to second-kind Fredholm integral equations.

1. Introduction. In this paper we derive convergence estimates for the iterative solution of nonsymmetric linear systems by GMRES [**25**]. We work in the context of strongly convergent-collectively compact [**2**] sequences of approximations to linear compact fixed point problems. Our estimates are intended to explain the observations that the performance of GMRES is independent of the discretization if the resolution of the discretization is sufficiently good. Our bounds are independent of the righthand side of the equation, reflect the r-superlinear convergence of GMRES in the infinite dimensional setting, and also allow for more than one implementation of the discrete scalar product. This latter property is important in the context of integral equations, where an integral operator may be discretized with a high-order quadrature rule, with the implicit approximation of the L^2 inner product by that quadrature rule, and GMRES implemented in software that uses the

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