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COUPLED VOLTERRA EQUATIONS WITH BLOW-UP SOLUTIONS

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ABSTRACT. A pair of coupled nonlinear Volterra equations are examined for possible blow-up solutions. The system is motivated by certain models of explosion phenomena in a diffusive medium. Criteria for a blow-up to occur as well as bounds on the time of its occurrence are derived for a general class of nonlinearities. Specific results are obtained for two special cases involving power law and exponential nonlinearities. Also, the asymptotic growth rate near blow-up is determined for these two special cases when the kernel behaves like that of the one-dimension heat equation.

1. Introduction. We examine a pair of coupled nonlinear Volterra equations, which are motivated by certain models of a diffusive medium that can experience explosive behavior. The particular models of interest are described by the vector integral equation,

$$u(t) = Tu(t) \equiv \int_0^t k(t-s)F[u(s)+h(s)]\,ds, \qquad t \geq 0, \tag{1}$$

where the components of the solution $u = [u_1, u_2]$ and the given data $h = [h_1, h_2]$ are real functions. The nonlinear operator $T = [T_1, T_2]$ is defined by the real components of the function $F = [F_1, F_2]$ and the real scalar kernel $k(t-s)$. The kernel is taken to be continuously differentiable and have the properties

$$k(t-s) \geq 0, \qquad k'(t-s) < 0, \qquad t > s \geq 0, \tag{2}$$

which are characteristic of problems involving diffusion.

The arguments of the component functions F_j, $j = 1, 2$, are restricted so that

$$F_1 = F_1[u_2 + h_2], \qquad F_2 = F_2[u_1 + h_1]. \tag{3}$$

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