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STABILITY OF CONTINUOUS AND DISCRETE VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS BY LIAPUNOV APPROACH

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ABSTRACT. The Liapunov method is used to obtain stability conditions on some classes of Volterra integro-differential equations, and it is generalized for studying stability of discrete Volterra equations. As a first application of such generalization, stability conditions for the Volterra linear multistep methods, applied to a general linear system of Volterra integro-differential equations, are derived.

1. Introduction. The aim of this paper is stability investigation of Volterra integro-differential equations (VIDEs) and their discretized version.

Since universal stability conditions are not known, we should try to study stability problems for special classes of VIDEs under appropriate assumptions. These equations should also be treated as a collection of test problems which would be useful to check the stability of numerical methods.

As is known, a powerful tool to analyze stability of a problem is the direct Liapunov method. It consists of the use of suitable functionals depending on the solution of the equation under consideration. Through the construction of such functionals, in Section 2, we obtain stability conditions for different types of VIDEs. Some of these conditions, if the integral term is zero, coincide with the known stability conditions for ODEs.

In Section 3 we consider a discrete VIDE, that is, a difference equation of unbounded order. For this equation we prove a general theorem which gives stability conditions depending on the existence of suitable functions. In other words, we extend the discrete Liapunov theory to the case of unbounded order difference equations. We stress that this theorem furnishes, as in the continuous case, a powerful tool to

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