PERTURBATION ANALYSIS FOR SOME LINEAR BOUNDARY INTEGRAL OPERATORS

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ABSTRACT. In this paper we study the effect on a boundary integral operator when the surface of integration is modified. It turns out that under certain regularity assumptions the perturbed and the original operators converge to each other when the corresponding surfaces converge. The estimates are derived in the uniform operator norm. The proofs apply to the single and the double layer operator from potential theory and are extended to some more general linear operators. Our results are important for the error analysis of discretization schemes for integral equations using approximate surfaces like the panel method. Moreover, we demonstrate how these results can be applied for solving integral equations on domains with almost symmetries.

1. Motivation. Consider the integral equation

$$\lambda \rho + \mathcal{K}\rho = f$$

with a scalar λ and the linear integral operator

(2)
$$(\mathcal{K}\rho)(x) := \int_{\mathcal{B}} k(x,y) \, \rho(y) \, d\mathcal{B}(y), \qquad x \in \mathcal{B}$$

defined on a compact surface $\mathcal{B} \subset \mathbf{R}^3$. Here we assume that the kernel is of either one of the following types:

(3)
$$k(x,y) = \frac{g(|x-y|)}{|x-y|^{\alpha}} \quad \text{for} \quad \alpha < 2$$

or

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