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ON A SPECIAL CLASS OF NONLINEAR INTEGRAL EQUATIONS

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ABSTRACT. We consider nonlinear integral equations

$$\varphi(t, x(t)) + \int_0^1 \psi(t, s, x(t), x(s))\, ds = 0 \quad \text{for } t \in [0, 1]$$

with a certain monotonicity in the argument $y = x(t)$ and a certain compactness in the argument $x = x(s)$.

1. Introduction. In [**8**], the nonlinear integral equation

$$(Fx)(t) := x^2(t) - t^2 + \frac{J}{4\pi} \int_0^1 k(t, s, x(t), x(s))\, ds = 0 \tag{1.1}$$

was investigated. Here $J \geq 0$ is a parameter, $t \in [0, 1]$, and the kernel k is nonlinear in all four arguments with $\partial_y k(t, s, y, x) \geq 0$; cf. (3.11) below for the explicit form of k. Since k is $x(t)$-dependent in a nontrivial way, (1.1) is not of standard type. For example, the theory of Hammerstein or Volterra equations, cf. e.g. [**9, 10, 20, 21, 22**] or [**6**], does not include (1.1).

In fact integral equations which show the same characteristic features (described more precisely later on) as (1.1), and whose general form is

$$\varphi(t, x(t)) + \int_0^1 \psi(t, s, x(t), x(s))\, ds = 0 \quad \text{for } t \in [0, 1], \tag{1.2}$$

arises in a variety of other settings, e.g., in neutron transport theory; cf. [**16**] and [**2**]. As a result, it is of interest to obtain a general existence theory for equation (1.2).

Now the main observation concerning (1.1) is that a difference can be made between x with argument t and x with argument s. To make

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