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ASYMPTOTIC BEHAVIOR AT INFINITY OF SOLUTIONS OF MULTIDIMENSIONAL SECOND KIND INTEGRAL EQUATIONS

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ABSTRACT. We consider second kind integral equations of the form $x(s) - \int_\Omega k(s,t)x(t)\,dt = y(s)$ (abbreviated $x - Kx = y$), in which Ω is some unbounded subset of $\mathbf{R}^n$. Let X_p denote the weighted space of functions x continuous on $\overline{\Omega}$ and satisfying $x(s) = O(|s|^{-p})$, $s \to \infty$. We show that if the kernel $k(s,t)$ decays like $|s-t|^{-q}$ as $|s-t| \to \infty$ for some sufficiently large q (and some other mild conditions on k are satisfied), then $K \in B(X_p)$ (the set of bounded linear operators on X_p), for $0 \le p \le q$. If also $(I-K)^{-1} \in B(X_0)$, then $(I-K)^{-1} \in B(X_p)$ for $0 \le p < q$, and $(I-K)^{-1} \in B(X_q)$ if further conditions on k hold. Thus, if $k(s,t) = O(|s-t|^{-q})$, $|s-t| \to \infty$, and $y(s) = O(|s|^{-p})$, $s \to \infty$, the asymptotic behavior of the solution x may be estimated as $x(s) = O(|s|^{-r})$, $|s| \to \infty$, $r := \min(p,q)$. The case when $k(s,t) = \kappa(s-t)$, so that the equation is of Wiener-Hopf type, receives especial attention. Conditions, in terms of the symbol of $I-K$, for $I-K$ to be invertible or Fredholm on X_p are established for certain cases (Ω a half-space or cone). A boundary integral equation, which models three-dimensional acoustic propagation above flat ground, absorbing apart from an infinite rigid strip, illustrates the practical application and sharpness of the above results. This integral equation models, in particular, road traffic noise propagation along an infinite road surface surrounded by absorbing ground. We prove that the sound propagating along the rigid road surface eventually decays with distance at the same rate as sound propagating above the absorbing ground.

1. Introduction. We consider integral equations of the form

$$x(s) - \int_\Omega k(s,t)x(t)\,dt = y(s), \qquad s \in \overline{\Omega}, \tag{1.1}$$

where Ω is some unbounded open subset of $\mathbf{R}^n$, dt is n-dimensional Lebesgue measure and $x, y \in X$, the Banach space of bounded and continuous functions on $\overline{\Omega}$. We abbreviate (1.1) in operator form as

$$x - Kx = y \tag{1.2}$$

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