JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 7, Number 2, Spring 1995

A DISCRETE COLLOCATION METHOD FOR BOUNDARY INTEGRAL EQUATIONS

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ABSTRACT. We propose a discrete collocation method for the boundary integral equations which arise from solving Laplace's equation $\Delta u = 0$. The Laplace's equation is defined on connected regions D in $\mathbf{R}^3$ with a smooth boundary S. The piecewise polynomial interpolation in the parametrization variables along with the collocation method is used, and a numerical integration scheme for collocation integrals is given. We give an estimation on the rate of convergence and present some numerical examples for the exterior Neumann problem.

1. Introduction. We propose a discrete collocation method for the boundary integral equations of the second kind for solving Laplace's equation $\Delta u = 0$ on connected regions D in $\mathbf{R}^3$. The integral equations we considered have the following form:

$$2\pi\rho(P) + \int_S \rho(Q)\frac{\partial}{\partial\nu_Q}\left[\frac{1}{|P-Q|}\right] dS_Q = g(P), \qquad P \in S. \tag{1.1}$$

Symbolically, we rewrite the integral equation (1.1) as

$$(2\pi + \mathcal{K})\rho = g$$

where $\mathcal{K} : C(S) \to C(S)$ defined by

$$\mathcal{K}\rho(P) = \int_S \rho(Q)\frac{\partial}{\partial\nu_Q}\left[\frac{1}{|P-Q|}\right] dS_Q$$

is a bounded compact linear operator.

To see where the above integral equations may arise in solving Laplace's equation, consider the following two problems:

Received by the editors on March 7, 1994, and in revised form on January 25, 1995.
Key words. Numerical integration, quadratic interpolation, adaptive refinement, collocation method.
AMS MOS *Subject Classification.* 65D30, 65D32, 65R20.