

**HETEROCLINIC ORBITS AND CONVERGENCE
OF ORDER-PRESERVING SET-CONDENSING
SEMIFLOWS WITH APPLICATIONS TO
INTEGRODIFFERENTIAL EQUATIONS**

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ABSTRACT. Several theorems on order-preserving set-condensing semiflows are proved. These results are then applied to a model of stage-structured populations with dispersal between patches in a heterogeneous environment.

1. Introduction. In [15], Smith proved that a cooperative and irreducible retarded functional differential equation with finite delay generates an eventually strongly monotone semiflow to which the powerful theory of monotone dynamical systems developed by Hirsch [5, 6] and Matano [7, 8] as well as the spectral theory of positive operators established by Nussbaum [10, 11] can be applied. Results in [15] have later been extended to more general retarded or neutral equations with finite delay [16, 20] and some integrodifferential equations with certain specific kernels [18, 19]. However, as an example in [18] indicates, solutions of integrodifferential equations with general kernels which satisfy the usual quasimonotonicity and irreducibility conditions always coincide with their initial values, and hence the solution semiflows can never be (eventually) strongly monotone if the state space consisting of some functions defined on the noncompact interval $(-\infty, 0]$ is endowed with the natural pointwise ordering. It is therefore natural to ask to what extent Smith's results can be generalized to cooperative and irreducible integrodifferential equations with general kernels and, more generally, to what extent the strong monotonicity or strong order-preserving conditions in the theory of monotone dynamical systems due to Hirsch and Matano can be relaxed.

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