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# TWO-GRID METHODS FOR NONLINEAR MULTI-DIMENSIONAL WEAKLY SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. The convergence rate of the piecewise constant collocation method for the nonlinear weakly singular integral equation is investigated by G. Vainikko [**8**]. For this method, it is necessary to solve a large nonlinear algebraic system. This can be done straightforwardly only for comparatively rough discretizations. In this paper a two-grid iteration method is considered which enables us to find practically the solution of this system for fine discretizations. The main result is Theorem 3 about the convergence and the convergence rate of this method. This theorem generalizes for nonlinear equations the result proved in [**7, 8**] for linear equations.

**1. Integral equation.** In this paper we shall deal with the integral equation

$$u(x) = \int_G K(x,y,u(y))\,dy + f(x), \qquad x \in G, \tag{1}$$

where $G \subset \mathbf{R}^n$ is an open bounded set with a piecewise smooth boundary $\partial G$. The following assumptions (A1)–(A4) are made.

(A1) The kernel $K(x,y,u)$ is twice continuously differentiable with respect to $x$, $y$ and $u$ for $x \in G$, $y \in G$, $x \neq y$, $u \in (-\infty,\infty)$, whereby there exists a real number $\nu \in (-\infty, n)$ such that, for any nonnegative integer $k \leq 2$ and $\alpha = (\alpha_1,\dots,\alpha_n) \in \mathbf{Z}_+^n$, $\beta = (\beta_1,\dots,\beta_n) \in \mathbf{Z}_+^n$ with $k + |\alpha| + |\beta| \leq 2$ the following inequalities hold:

$$|D_x^\alpha D_{x+y}^\beta \frac{\partial^k}{\partial u^k} K(x,y,u)| \leq b_1(|u|)\gamma_{\nu+|\alpha|}(x,y),$$

$$\begin{aligned}|D_x^\alpha D_{x+y}^\beta \frac{\partial^k}{\partial u^k} K(x,y,u_1) - D_x^\alpha D_{x+y}^\beta \frac{\partial^k}{\partial u^k} K(x,y,u_2)|\\ \leq b_2(\max\{|u_1|,|u_2|\})|u_1-u_2|\gamma_{\nu+|\alpha|}(x,y).\end{aligned}$$

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