A WAVELET ALGORITHM FOR THE SOLUTION OF THE DOUBLE LAYER POTENTIAL EQUATION OVER POLYGONAL BOUNDARIES

ANDREAS RATHSFELD

ABSTRACT. In this paper we consider a piecewise linear collocation method for the solution of the double layer potential equation corresponding to Laplace's equation over polygonal domains. We give a wavelet algorithm for the computation of the corresponding stiffness matrix and for the solution of the arising matrix equation with no more than $O(N \cdot [\log N]^8)$ arithmetic operations. The error of the resulting approximate solution is of order $O(N^{-2} \cdot [\log N]^6)$. Finally, we give some remarks on the generalization of the algorithm to the piecewise cubic collocation and present numerical tests.

0. Introduction. The most popular numerical methods for the approximate solution of boundary value problems for elliptic partial differential equations are finite difference or finite element methods. However, there is a well-known alternative, the so-called boundary element method. Following this scheme, one reduces the boundary value problem for the differential equation over a given domain to a certain integral equation over the boundary of the domain. Substituting the solution of this integral equation into an integral representation formula yields the solution of the original partial differential equation. The advantages of this method in comparison to finite element or finite difference schemes consist in the facts that the approximate solution fulfills the partial differential equation exactly (of course, the boundary conditions hold only approximately) and that the discretization of the boundary is often simpler than that of the domain (in particular, the discretization of the boundary is easier if the domain is unbounded). Another advantage should be the reduction of the dimension of the problem. In fact, if the partial differential equation is to be solved over a d dimensional domain, then the boundary integral equation is defined over a $d - 1$ dimensional boundary manifold. Consequently, the linear systems of equations which arise after the discretization