JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 7, Number 1, Winter 1995

REDUCTION OF AN INFINITE SYSTEM OF INTEGRAL EQUATIONS OF POTENTIAL TYPE ON A ONE-DIMENSIONAL LATTICE OF CLOSED CURVES IN THE PLANE TO A FINITE SYSTEM OF INDEPENDENT PSEUDODIFFERENTIAL EQUATIONS ON A CIRCLE

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ABSTRACT. An infinite system of integral equations which arises in the reduction of the Dirichlet problem for the Helmholtz equation in the plane to the boundary is considered. The boundary is formed by an infinite network of nonintersecting infinitely smooth simple closed curves obtained from a fixed one by a parallel translation by vectors belonging to a one-dimensional lattice. It is shown that if the righthand side of the system is a T-periodic function on the lattice, then the system can be reduced to a system of T independent pseudodifferential equations on the unit circle with classic elliptic pseudodifferential operators of order -1 in the Sobolev scale. Another significant outcome of this work is that this reduction allows one to apply the many known powerful methods for the numerical analysis of classic elliptic pseudodifferential equations on the unit circle to the original system.

1. Introduction. It is well known (see, for example, [**1**–**3**]) that the Dirichlet boundary value problem for the Helmholtz equation in the plane, with radiation condition at infinity and the boundary formed by a finite number N of nonintersecting infinitely smooth simple closed curves, can be reduced to a system of N integral equations. The $N \times N$ matrix integral operator of this system was proved to be an $N \times N$ matrix classic elliptic pseudodifferential operator of order -1 in the Sobolev scale of N-dimensional complex vector functions on the unit circle. The practical benefit of viewing the original system of integral equations as a system of pseudodifferential equations is that one can then apply the many known powerful methods for the

Received by the editors on September 8, 1994.
1991 *Mathematics Subject Classification.* Primary 45F15, 35S99, 31A10, Secondary 35J05, 35B27.
Key words and phrases. System of integral equations, pseudodifferential equations, potential theory, Helmholtz equation, Dirichlet problem.