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A BOUNDARY ELEMENT METHOD FOR A NONLINEAR BOUNDARY VALUE PROBLEM IN STEADY-STATE HEAT TRANSFER

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ABSTRACT. A new boundary variational formulation is presented for the steady-state heat equation with radiation, and a boundary element method is presented for its solution. Furthermore, an optimal error estimate for the boundary element approximation is given.

1. Introduction. We consider the numerical modeling of steady-state heat transfer with radiation. This phenomenon is mathematically represented by the following nonlinear boundary value problem

$$\Delta u = 0, \quad \text{in } \Omega, \tag{1}$$

$$\frac{\partial u}{\partial n} + \gamma |u|^3 u = f(x), \quad \text{on } \Omega, \tag{2}$$

where $\Omega \subset R^2$ is a bounded domain with sufficient smooth boundary Γ, γ is a positive constant associated with the body's emittance [**15**], and f is a given function on Γ. In the engineering literature, the nonlinear boundary value problem (1)–(2) was studied by several authors using boundary element methods [**5, 9, 14**]. Recently Ruotsalainen and Wendland [**13**] have considered the potential problem with boundary condition

$$\frac{\partial u}{\partial n} + g(x,u) = f(x), \quad \text{on } \Gamma, \tag{3}$$

and have given an analysis of a boundary element method for problem (1)–(3). They assume $g : \Gamma \times R \to R$ is a Caratheodory-function, i.e.,

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