JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 7, Number 1, Winter 1995

RUNGE-KUTTA TIME DISCRETIZATIONS OF PARABOLIC VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we prove optimal convergence results for implicit Runge-Kutta methods applied to parabolic Volterra integro-differential equations in Hilbert spaces. For this purpose, we show that such equations can be rewritten as parabolic differential equations in appropriate function spaces. Since Runge-Kutta methods are invariant under this transformation, it is sufficient to show that these methods converge for the resulting differential equations. While our theoretical framework is applicable to general (nonlinear) Volterra integro-differential equations, it is realized here in detail for the linear case.

1. Introduction. The aim of this paper is the study of the approximation properties of implicit Runge-Kutta methods for the abstract linear Volterra integro-differential equation (VIDE)

$$y'(t) + A(t)y(t) = f(t) + \int_0^t B(t,\tau)y(\tau)\,d\tau, \qquad 0 < t \leq T, \tag{1.1}$$

subject to the initial condition $y(0) = y_0$ in a Hilbert space. We are mainly interested in the case where $A(t)$ and $B(t,\tau)$ are differential operators. More precise conditions will be given in Section 3 where we will specify the function spaces which form the framework of our analysis.

Abstract VIDEs of the general form (1.1) or of convolution type,

$$y'(t) + Ay(t) = f(t) + \int_0^t B(t-\tau)y(\tau)\,d\tau, \tag{1.2}$$

Received by the editors on June 1, 1994, and in revised form on December 6, 1994.
Key words and phrases. Integro-differential equation, parabolic problem, Runge-Kutta method, fractional order of convergence.
1991 *Mathematics Subject Classification.* 65R20, 65J10, 45K05, 45L10.