

**ASYMPTOTIC BEHAVIOR OF THE SOLUTIONS  
OF THE INTEGRO-DIFFERENTIAL EQUATIONS  
ON POSITIVE HALF-AXIS WITH NON-DIFFERENCE  
KERNEL OF A CERTAIN TYPE**

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**ABSTRACT.** The asymptotic behavior of the solutions of a class of integro-differential equations on positive half-axis with non-difference kernel is investigated. The solutions are equal asymptotically to the sum of terms having the form of product of an exponent  $e^{p_k x}$  and a polynomial. Numbers  $p_k$  are zeros of a function which is found in explicit form. Location of these zeros in the complex plane is also investigated. To obtain these results the technique of analytical continuation is used.

**1. Introduction. Formulation of the problem, and the main result.** There are many applied problems which lead to the equations of the form:

$$(1) \quad -\frac{d^2 y}{dx^2} + y = \int_0^\infty R(x-t)y(t) dt + \int_0^\infty R_1(x+t)y(t) dt, \quad x > 0.$$

Equations of this kind arise in various fields of physics. As such, we may mention radiative equilibrium of stars [3], anomalous skin-effect in metals [8,4,2] stationary neutron density in multiplying media [1, 5], wave propagation in acoustic and electrodynamic waveguides [10, 11, 7, 9]. In all these fields of research there are many particular problems and cases which lead to the equation (1) with  $R_1 \equiv 0$ . These cases have been exhaustively treated with the standard Wiener-Hopf technique. However, there are many problems which cannot be simplified in this way. That is why the equation (1) in its general form deserves an independent investigation. It turned out rather unexpectedly that in many cases the results obtained in this paper as far as asymptotic behavior is concerned justify replacing of equation (1) with the equation with kernel  $R_1 \equiv 0$ . However, as is often the case, the more subtle features of the solution essentially depend on the kernel  $R_1$ . As an

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Received by the editors on March 20, 1994, and in revised form on September 22, 1994.

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