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FULLY-DISCRETE COLLOCATION METHODS FOR AN INTEGRAL EQUATION OF THE FIRST KIND

WILLIAM MCLEAN

ABSTRACT. Using a model boundary integral equation of the first kind, we study some very simple numerical integration schemes for implementing spline collocation methods. The logarithmic singularity in the kernel is handled by combining special correction terms with standard composite integration rules of Gauss or Lobatto type. We prove that the stability and asymptotic convergence properties of the collocation method are maintained despite the quadrature errors. Numerical experiments confirm the error analysis.

1. Introduction. Consider the logarithmic-kernel integral equation of the first kind,

$$\int_\Gamma U(Y)\log\frac{\omega}{|X-Y|}\,ds_Y = F(X) \quad \text{for } X\in\Gamma. \tag{1.1}$$

Here Γ is a smooth, closed curve in the plane, $|X-Y|$ is the Euclidean distance between the points X and Y, and ds_Y is the element of arc length at Y. (The role of the parameter ω is explained below.) A standard numerical technique for solving boundary integral equations such as (1.1) is the collocation method. In this paper we use the theory in [**8**] to design some new and very simple numerical integration techniques for handling the integrals that define the entries of the collocation matrix. The resulting fully-discrete methods exhibit the same rates of convergence as would be achieved using the collocation method with exact integration.

Symm's equation arises in boundary integral reformulations of the Dirichlet problem for the Laplace equation in two dimensions, see [**7**]. Other second-order elliptic partial differential equations lead to similar integral equations of the first kind, with kernels that have the same

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