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ON THE EXISTENCE OF A GLOBAL MILD SOLUTION FOR A NONLINEAR INTEGRODIFFERENTIAL EQUATION WITH A SINGULAR KERNEL

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ABSTRACT. We study the equation

$$u_t(t,x) = \int_0^t b(t-s)g(u_x(s,x))_x\,ds + f(t,x),$$

where u is the unknown and the kernel b, the nonlinear function g, and the forcing function f are given. We assume that the kernel $b:(0,\infty)\to R$ is nonnegative, nonincreasing, convex and singular.

Making use of energy estimates and the theory of maximal monotone operators, we prove the global existence of a mild solution.

1. Introduction. We consider the equation

$$u_t(t,x) = \int_0^t b(t-s)g(u_x(s,x))_x\,ds + f(t,x); \qquad t>0,\ 0\le x\le 1, \tag{1.1}$$

with the boundary and initial conditions

$$u(t,0) = u(t,1) = 0; \qquad t>0,$$
$$u(0,x) = u_0(x); \qquad 0\le x\le 1.$$

Here u is the unknown and the kernel b, the nonlinear function g, the forcing function f, and the initial data u_0 are given. Under certain assumptions, the equation (1.1) models the behavior of a thin viscoelastic body (see [**8, 11, 15**]).

We study the equation (1.1) with a singular kernel, that is, we assume that $b\in L^1_{\mathrm{loc}}([0,\infty))$ and that $\lim_{t\to 0+} b(t)=\infty$. While the singularity

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