JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 6, Number 4, Fall 1994

A STABILITY THEORY FOR INTEGRAL EQUATIONS

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1. Introduction. In large measure, stability theory of differential equations centers around equilibrium points, either those occurring naturally in the equation or constructed by a change of variable. Thus, if we are interested in a stability theory for integral equations, then we need to decide just what will play the role of an equilibrium point. This is particularly important if we wish to employ Liapunov functions because they are constructed so as to be positive definite with respect to an equilibrium point.

In this paper we offer one choice for equilibrium points and we show that it is a good choice by developing a Liapunov theory around it and use it to obtain new results on limit sets for three problems of classical interest.

In particular, we study three forms of the integral equation

$$x(t) = a(t) - \int_{\alpha(t)}^{t} Q(t, s, x(s))\, ds \tag{1}$$

where $\alpha(t) \geq \alpha \geq -\infty$. We focus on functions which are analogous to equilibrium points of ordinary differential equations and obtain results, by way of Liapunov's direct method, concerning the long-time behavior of solutions.

Definition 1. A pair of functions (ψ, Ψ), each mapping $[\alpha, \infty) \to R^n$ with $\alpha \leq 0$, is said to be a *near equilibrium* for (1) if

$$\Psi(t) := a(t) - \psi(t) - \int_{\alpha(t)}^{t} Q(t, s, \psi(s))\, ds \in L^1[0, \infty). \tag{2}$$

Thus, if (1) is perturbed by the L^1 function Ψ, then ψ is a solution of (1); in other words, ψ fails to be a solution of (1) by an amount of an L^1 function.

Received by the editors on February 23, 1994.
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