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ASYMPTOTIC PROPERTIES VIA AN INTEGRODIFFERENTIAL INEQUALITY

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ABSTRACT. Recent results about asymptotic properties for integrodifferential equations in $\Re^n$ are studied in Hara, et al. [**6**] by analyzing a Liapunov function $v(\cdot)$ satisfying

$$v'(t) \leq -\alpha v(t) + \int_0^t \omega(t,s)v(s)\,ds,$$
$$t \geq t_0 \geq 0.$$

We will extend the techniques in [**6**] to the study of integrodifferential equations

$$x'(t) = A(t)\left[x(t) + \int_{\#}^t F(t,s)x(s)\,ds\right],$$
$$t \geq t_0 \geq 0,\ (\# = 0 \text{ or } -\infty),$$

in real Hilbert spaces with unbounded linear operators $A(\cdot)$, when a Liapunov function $v(\cdot)$ satisfies

$$v'(t) \leq -\alpha v(t) + \sqrt{v(t)} \int_{\#}^t \omega(t,s)\sqrt{v(s)}\,ds,$$
$$t \geq t_0 \geq 0\ (\# = 0 \text{ or } -\infty).$$

The results include uniform stability and asymptotic stability, as well as uniform boundedness and ultimate boundedness, which are not studied in [**6**]. The above integrodifferential equations occur in viscoelasticity and in heat conduction for materials with memory.

1. Introduction. In qualitative studies of differential or integrodifferential equations, Liapunov or Liapunov-Razumikhin methods are

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