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EXISTENCE, UNIQUENESS AND SMOOTHNESS RESULTS FOR SECOND-KIND VOLTERRA EQUATIONS WITH WEAKLY SINGULAR KERNELS

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ABSTRACT. We consider second-kind Volterra equations with weakly singular kernels. When the kernels assume simple forms, we find analytic solution expressions and prove existence, uniqueness and smoothness properties. Similar results for some general cases are then proved by using an idea of Professor Kendall Atkinson [**1**].

1. Introduction. The purpose of this paper is to study second-kind Volterra equations with weakly singular kernels of the forms

$$y(t) + \int_0^t K(t,s)p(t,s)y(s)\,ds = f(t), \qquad t \in (0,T] \tag{1.1}$$

and

$$y(t) - \int_0^t K(t,s)q(t,s)y(s)\,ds = g(t), \qquad t \in (0,T]. \tag{1.2}$$

In these equations the weakly singular kernels are expressed as the product of a smooth part, $K(t,s)$, and a singular part, $p(t,s)$ or $q(t,s)$, with

$$p(t,s) = \frac{1}{\sqrt{\pi}}\,\frac{1}{\sqrt{\ln(t/s)}}\left(\frac{s}{t}\right)^{\mu}\frac{1}{s}$$

$$q(t,s) = \left(\frac{s}{t}\right)^{\mu}\frac{1}{s}$$

for some $\mu > 0$.

Special cases of the equations of the types (1.1) and (1.2) arise from certain practical applications (cf. [**2, 6**]). The case when the smooth

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