JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 6, Number 3, Summer 1994

VARIATIONAL METHOD WITH APPLICATION TO CONVOLUTION EQUATIONS

DANIEL GOELEVEN

ABSTRACT. The aim of this paper is to solve the convolution equation $k * u^2 = |u|$ for k subject to the conditions $k \in L^{3/2}(\mathbf{R})$, $k(x) \geq 0$, $k(x) = k(-x)$ and k symmetrically decreasing. By using a result of the Ljusternik-Schnirelman theory on C^1-manifold due to A. Szulkin we improve some recent results of J.B. Baillon and M. Théra.

1. Introduction. Recently, J.B. Baillon and M. Théra [**1**] introduced a notion of self-adjoint nonlinear operator T with respect to a duality mapping J_θ. Using the properties of such a mapping they studied the optimization problem

$$\text{(P)} \qquad \max\{\langle Tu, J_\theta u\rangle : u \in X, ||u|| = 1\},$$

where X is a reflexive real Banach space equipped with a sufficiently smooth norm.

In their papers [**1, 2, 12**], they showed that problem (P) was very useful to obtain solutions of some convolution equations such as the following:

$$\text{(E)} \qquad k * u^2 = u, \qquad u \in L^3(\mathbf{R}),$$

where $k \in L^{3/2}(\mathbf{R}) \cap L^3(\mathbf{R})$ is assumed to be symmetrically decreasing, even and positive.

In this paper we use a recent result of the Ljusternik-Schnirelman theory on C^1-manifold [**10**] due to A. Szulkin to find critical points of the norm $|| \cdot ||$ on the Banach manifold $M := \{u \in X : \langle Tu, J_\theta u\rangle = 1\}$. By using the Lagrange multiplier theorem we prove that our approach can also be used to find solutions of convolution equations and of some set-valued integral equations.

Received by the editors on September 21, 1993.
AMS *Mathematics Subject Classification.* 45G05, 45G10, 49B99.