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FAST NUMERICAL SOLUTION OF SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. We study a fast method for the numerical solution of Cauchy singular integral equations. The method is based on fast inversion of the principal part, and on multi-grid methods for the resulting Fredholm integral equation of the second kind. The inversion of the principal part is done via an approximate Wiener-Hopf factorization using FFTs. Under suitable smoothness assumptions, the algorithm requires $O(n \log n)$ operations to achieve an accuracy comparable to that of the trigonometric collocation method with n collocation points, which is known to give quasi-optimal approximations.

1. Introduction. We consider fast algorithms for the approximate solution of Cauchy integral equations over closed curves in the plane. We may write such equations as

$$c(\zeta)u(\zeta) + \frac{d(\zeta)}{\pi i}\int_\Gamma \frac{u(z)}{z-\zeta}\,dz + \frac{1}{2\pi i}\int_\Gamma k(\zeta, z)u(z)\frac{dz}{z} = f(\zeta) \tag{1.1}$$

over the unit circle in the complex plane $\Gamma = \{\zeta \in \mathbf{C} : |\zeta| = 1\}$, with integration direction counterclockwise. We will assume that c, d, k and f are fairly smooth functions. We write (1.1) in operator notation as

$$(A + K)u = f, \tag{1.2}$$

where $A = cI + dS$, with S denoting the singular integral operator, and where K is the above integral operator with kernel k. The operator $A + K$ can be invertible in its natural setting only if A is invertible, which is why A is called the principal part. The algorithm proposed in the present paper is actually a discretization of the "preconditioned" equation, a Fredholm integral equation of the second kind:

$$(I + A^{-1}K)u = A^{-1}f. \tag{1.3}$$

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