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SINGULARITY PRESERVING GALERKIN METHODS FOR WEAKLY SINGULAR FREDHOLM INTEGRAL EQUATIONS

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ABSTRACT. Singularity preserving projection methods are developed in this paper for Fredholm integral equations of the second kind with weakly singular kernels. These methods give an optimal order of convergence for the approximate solutions. As an application, the singularity preserving Galerkin approximation for equations with logarithmic or algebraic singular kernels is discussed in detail. This is done by deriving singularity expansions for the solutions of these equations. A numerical example is given to illustrate the error estimates.

1. Introduction. In the last decade there has been considerable interest in the numerical analysis of solutions of integral equations with weakly singular kernels. Most of the existing numerical methods for these equations concentrate on approximating the solutions by functions without singularities, e.g., by polynomials or splines. In this work we establish Galerkin approximations that preserve the singularities of the solutions and possess an optimal order of convergence. This will be done by allowing the projection subspaces to contain some known singular functions that carry the singularities of the exact solutions. The singularities of the approximate solutions will cancel with those of the exact solutions, and consequently, the order of convergence will achieve the optimal rate. The regularity properties and singularity expansions of the solutions play a central role in this work.

Let $L_p = L_p[0,1]$ be the Banach space of pth power integrable functions with norm defined by $||f||_p = (\int_0^1 |f(t)|^p\,dt)^{1/p}$ for $1 \le p < \infty$. In this paper we study singularity preserving projection methods for solutions of Fredholm integral equations of the second kind that take the form

$$y(s) - \int_0^1 k(s,t)y(t)\,dt = f(s), \qquad 0 \le s \le 1, \tag{1.1}$$

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