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PSEUDOSPECTRA AND SINGULAR VALUES OF LARGE CONVOLUTION OPERATORS

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ABSTRACT. This paper is an introduction to C^*-algebra methods for studying the spectral behavior of large truncated Wiener-Hopf operators. We compute the limit of the norms of the inverses of truncated Wiener-Hopf operators and we show that the pseudospectra (in contrast to the spectra) and the singular values of large truncated Wiener-Hopf operators mimic the pseudospectrum and the singular values of the original operator.

1. Introduction. Given a function $k \in L^1(\mathbf{R})$ and a finite interval $(\alpha, \beta) \subset \mathbf{R}$, we consider the convolution operator on $L^2(\alpha, \beta)$ defined by

$$(W_{\alpha,\beta}\varphi)(x) = \int_\alpha^\beta k(x-t)\varphi(t)\,dt, \qquad \alpha < x < \beta.$$

What can be said about the spectrum

$$\Lambda_0(W_{\alpha,\beta}) := \{\lambda \in \mathbf{C} : W_{\alpha,\beta} - \lambda I \text{ is not invertible}\}$$

if $\tau = \beta - \alpha$ is a large number? In that case one might try one's luck by replacing $W_{\alpha,\beta}$ with the operator M given on $L^2(-\infty, \infty)$ by

$$(M\varphi)(x) = \int_{-\infty}^\infty k(x-t)\varphi(t)\,dt, \qquad -\infty < x < \infty,$$

or since clearly $\Lambda_0(W_{\alpha,\beta}) = \Lambda_0(W_{0,\tau})$, one could also substitute for $W_{\alpha,\beta}$ the operator W acting on $L^2(0,\infty)$ by the rule

$$(W\varphi)(x) = \int_0^\infty k(x-t)\varphi(t)\,dt, \qquad 0 < x < \infty.$$

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