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DISCRETE POLYNOMIAL-BASED GALERKIN METHODS FOR FREDHOLM INTEGRAL EQUATIONS

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1. Introduction. In recent years there has been considerable interest in using Galerkin's method for the numerical solution of Fredholm integral equations. In large measure, this interest seems to stem from the interesting superconvergence properties discovered by Sloan in [**12, 13**]. In early work the effect of quadrature errors on the behavior of the algorithms was ignored [**12, 13**], but starting with the work of Chandler [**4**] and then Spence and Thomas [**14**], these errors have been studied in great detail. For spline approximations this work culminated in the papers of Joe [**7**] and Atkinson and Bogomolny [**2**]. In particular, in [**2**], it was shown that sufficiently accurate quadrature rules preserved both the rates of convergence and superconvergence of Galerkin's method.

In [**5**] Delves and Freeman discussed the effect of quadrature errors on Galerkin's method using orthogonal polynomial approximations for one-dimensional equations, while Miel in [**10, 11**] examined the particular case of Legendre polynomial approximations for both linear and nonlinear equations. None of these authors considered the convergence of the Sloan iterate.

It is the purpose of this paper, therefore, to sharpen and extend the convergence results in [**5, 10, 11**] for the solution of the equations

$$u(x) = f(x) + \int_a^b k(x,t)u(t)\,dt, \quad -\infty < a < b < \infty, \tag{1.1}$$

where $f(x)$ and $k(x,t)$ are suitably smooth functions on $[a,b]$ and $[a,b]\times[a,b]$ respectively. In particular, we show that if $u(x)$ is approximated by $v_n = \sum_{k=0}^n a_k\varphi_k(x)$, where $\{\varphi_n\}$ are the orthonormal polynomials associated with the integrable weight function $w(x) \geq 0$ on $[a,b]$ and integration rules of precision greater or equal than $2n$ are used to evaluate the integral transforms and inner products, then $||u -$

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