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DIFFERENTIAL APPROXIMATION FOR VISCOELASTICITY

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1. Introduction. This paper is a continuation of work begun in [**4**]. The general area is that of evolution equations containing a hereditary (time nonlocal) effect which produces dissipation. The goal is to devise approximate equations which are easier to handle but which accurately reproduce dissipation.

The equation studied in [**4**] was "parabolic" in nature. Here we deal with the "hyperbolic" situation. We take as a model the displacement problem for linear, isotropic viscoelasticity. Let us describe the problem in order to motivate the ideas.

We let Ω be a bounded region in space representing a reference configuration for a body, and we let $\mathbf{u}(x,t)$ denote displacement. For ease of exposition, we assume the body is homogeneous. Let μ and λ be functions of t on $[0,\infty)$. We write:

$$\begin{aligned} E[\mathbf{u}] &= (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2 \\ L(\mu,\lambda)[\mathbf{u}] &= 2\mu E[\mathbf{u}] + \lambda \operatorname{tr} E[u]\mathbf{I}. \end{aligned} \tag{1.1}$$

Then linear, isotropic viscoelasticity (for a solid) is described by giving the stress $\sum(x,t)$ by the formula, [**7, 8**],

$$\sum(x,t) = \frac{\partial}{\partial t}\int_{-\infty}^{t} L(\mu(t-\tau),\lambda(t-\tau))[\mathbf{u}(x,\tau)]\,d\tau. \tag{1.2}$$

Remark. For an inhomogeneous material μ and λ are functions of x and t.

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