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CONTINUOUS SOLUTIONS OF A NONLINEAR INTEGRAL EQUATION ON AN UNBOUNDED DOMAIN

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ABSTRACT. The existence of bounded continuous solutions to the integral equation

$$z(t) = z_0(t) + f\,(t, z(t)) \int_I g\,(t, s, z(s))\,ds \quad t \in I\,,$$

where I is an unbounded closed real interval, is established. This is achieved by means of results concerning α-set contractions. Equations of this type arise in the theories of radiative transfer, neutron transport and in the kinetic theory of gases.

1. Introduction. Integral equations often arise in many physical or chemical problems. If the equation involves only compact or linear integral operators, numerous existence results are available [**4, 11**]. Most of them assume that the domain of the independent variable is bounded. When one attempts to solve integral equations on unbounded domains or containing noncompact terms, many difficulties arise and the classical methods (as, for instance, Schauder Fixed Point Theorem, Fredholm theory, ...) are not always applicable in a simple way. Some authors have overcome these difficulties by resorting to different techniques based on, for example, measures of noncompactness [**8, 9**] or on strict convergence in suitable function spaces [**1**].

In this paper we consider an integral equation containing the term $F\,K$, where F is a superposition operator and K a compact integral operator. The domain I of the unknown function involved in this equation is a closed real interval of infinite measure. We look for solutions that are continuous and bounded in I.

The most famous equation with a term of this type is the Boltzmann equation, which describes the evolution of a gas in the framework of the kinetic theory. In fact, consider a simple gas whose molecules are

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