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THE JACOBI METHOD IN WEIGHTED BANACH SPACES FOR INTEGRAL EQUATIONS, WITH EMPHASIS ON GREEN'S-FUNCTION-LIKE KERNELS

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ABSTRACT. The theoretical foundations for the iterative solution of integral equations in weighted function spaces are derived which allow a unified treatment of the following two cases. Case 1: the kernel is nonzero on its whole domain of definition. Case 2: the kernel is nonzero in the interior of its domain of definition and zero or partly zero on the boundary, which often occurs with Green's functions. The method presented is new, particularly the theory of condensing operators according to [**10**, pp. 102–109] can be avoided, which simplifies the treatment of the corresponding integral equations considerably. Further, strong and weak convergence criteria are given. The results are used to sharpen error estimates in the Jacobi method (i.e., in the method of successive approximations) and are applied to a boundary value problem. Numerical tests show good agreement with the theoretical results.

0. Introduction. There are important relationships between the dynamic analysis of elastic structures in engineering science and the iterative methods of positive completely continuous operators in numerical mathematics. In dynamics, eigenfrequencies and eigenfunctions play a fundamental role. For many problems the first eigenfrequency and corresponding eigenfunction are most important. Often, it is not sufficient to know merely the first eigenfrequency. Also, for iteration methods for positive completely continuous operators, the greatest eigenvalue (i.e., the spectral radius) and the corresponding positive eigenfunction are of similar importance.

The significance of the spectral radius is widely known: Under appropriate conditions, an iteration process for general operator equations converges, if the spectral radius of the iteration operator is less than one. It is less known (or at least, it is less made use of) that the (positive) eigenfunction corresponding to the spectral radius of an integral

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