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BOUNDARY VALUE PROBLEMS FOR INTEGRO-DIFFERENTIAL EQUATIONS OF BARBASHIN TYPE

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ABSTRACT. In this paper we study linear integro-differential equations of Barbashin type, subject to appropriate boundary conditions. In particular, we show how to transform such equations to linear two-dimensional integral equations of second kind. This allows us to apply the classical Fredholm theory to obtain existence, uniqueness, and multiplicity results. Next, we apply a general fixed point principle in K-normed spaces to get more general existence results. Moreover, we employ weighted function spaces to cover the case of unbounded multiplicator functions. The abstract theorems are illustrated by means of an application to the problem of radiation propagation in physical systems.

0. Introduction. In this paper we study the solvability of linear integro-differential equations of Barbashin type, subject to certain boundary conditions. Boundary value problems for Barbashin or similar equations arise in the mathematical modeling of various phenomena of transport theory, e.g., in the propagation of radiation through the atmosphere of planets and stars [**6, 19, 21**], the transfer of neutrons through thin plates and membranes in nuclear reactors [**22**], and in several other transport problems [**1, 7, 12**]. Integro-differential equations of Barbashin type also occur in some fields of probability theory [**1, 11, 14**], in acoustic scattering theory [**6**], and in systems with substantially distributed parameters [**5**]. For all these problems, an adequate mathematical description leads rather naturally to the boundary value problem considered in this paper.

It is well-known [**9, 24**] that integro-differential equations of Barbashin type provide a "continuous analogon" to countable systems of ordinary differential equations. Likewise, the boundary value problems we are going to study below may be interpreted as "continuous analoga"

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