

GENERALIZED CONDITIONAL YEH-WIENER INTEGRALS AND A WIENER INTEGRAL EQUATION

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ABSTRACT. Let $Q = [0, S] \times [0, T]$ and let $h \in L_2(Q)$. In this paper we evaluate conditional Yeh-Wiener integrals of the type

$$E \left[\exp \left\{ \int_0^t \int_0^s \phi(\sigma, \tau, \int_0^\tau \int_0^\sigma h(u, v) dx(u, v)) d\sigma d\tau \right\} \mid \int_0^t \int_0^s h(u, v) dx(u, v) = \xi \right].$$

The method we use to evaluate these conditional integrals is first to define a sample path-valued conditional Yeh-Wiener integral and show that it satisfies a Wiener integral equation. We next obtain a series solution to this Wiener integral equation which we then use to evaluate the above conditional Yeh-Wiener integral.

1. Introduction. For $Q = [0, S] \times [0, T]$, let $C(Q)$ denote Yeh-Wiener space, i.e., the space of all real-valued continuous functions $x(s, t)$ on Q such that $x(0, t) = x(s, 0) = 0$ for every (s, t) in Q . Yeh [10] defined a Gaussian measure m_y on $C(Q)$ (later modified in [11]) such that as a stochastic process $\{x(s, t), (s, t) \in Q\}$ has mean $E[x(s, t)] \equiv \int_{C(Q)} x(s, t) m_y(dx) = 0$ and covariance $E[x(s, t)x(u, v)] = \min\{s, u\} \min\{t, v\}$. Let $C_w \equiv C[0, T]$ denote the standard Wiener space on $[0, T]$ with Wiener measure m_w . In [12], Yeh introduced the concept of the conditional Wiener integral of F given X , $E[F | X]$, and for the case $X(x) = x(T)$ obtained some very useful results including a Kac-Feynman integral equation.

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