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PSEUDOSPECTRA OF WIENER-HOPF INTEGRAL OPERATORS AND CONSTANT-COEFFICIENT DIFFERENTIAL OPERATORS

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ABSTRACT. A number $z \in \mathbf{C}$ is in the ε-pseudospectrum of a linear operator A if $||(zI - A)^{-1}|| \geq \varepsilon^{-1}$. In this paper, we investigate the ε-pseudospectra of Volterra Wiener-Hopf integral operators and constant-coefficient differential operators with boundary conditions at one endpoint for the interval $[0, b]$. We show that although the spectra of these operators are not continuous in the limit $b \to \infty$, the ε-pseudospectra are continuous as $b \to \infty$ for all $\varepsilon > 0$. These results are an extension of previous work on the pseudospectra of Toeplitz matrices.

1. Introduction. Let $\mathcal{H}$ be a Hilbert space with inner product $(\cdot, \cdot)$ and norm $||\cdot||$. Let $T : \mathcal{H} \to \mathcal{H}$ be a closed linear operator with domain $\mathcal{D}(T)$, spectrum $\Lambda(T)$, and resolvent set $\rho(T)$ [**9**]. For each $\varepsilon \geq 0$, the ε-pseudospectrum of T, which we denote by $\Lambda_\varepsilon(T)$, can be defined in the following manner [**21, 22**]:

Definition. For each $\varepsilon \geq 0$, a number $z \in \mathbf{C}$ is in the ε-pseudospectrum of T if

$$z \in \{\lambda \in \rho(T) : ||(\lambda I - T)^{-1}|| \geq \varepsilon^{-1}\} \cup \Lambda(T). \tag{1.1}$$

This definition is essentially equivalent to that for the set of ε-*approximate eigenvalues* introduced by Landau [**11**]. Similar sets have also been considered by other researchers; see [**21, 22**] for a discussion.

As the definition shows, the sets $\Lambda_\varepsilon(T)$ are nested and $\Lambda_0(T)$ is the spectrum. Pseudospectra were introduced by Trefethen [**20**] to analyze the behavior of *non-normal* matrices. A normal matrix satisfies $A^+A = AA^+$, where A^+ is the adjoint, and has orthogonal eigenfunctions. The ε-pseudospectrum of a normal matrix is simply the union of the closed

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