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LOCAL EXISTENCE FOR ABSTRACT SEMILINEAR VOLTERRA INTEGRODIFFERENTIAL EQUATIONS

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ABSTRACT. Local existence is proved for mild solutions of a linear Volterra equation of convolution type in a Banach space, perturbed by a continuous nonlinear hereditary term. The linear part, which involves an unbounded linear operator, has a resolvent kernel with certain compactness properties that permit one to use Schauder's theorem to obtain local existence for the perturbed equation without stronger conditions on the nonlinearity.

1. Introduction and statement of results. For the Volterra integrodifferential equation

$$\begin{aligned} \mathbf{x}'(t) &= \int_0^t a(t-\tau)\mathbf{L}\mathbf{x}(\tau)d\tau + \mathbf{f}(t), \quad t>0 \\ \mathbf{x}(0) &= \mathbf{x}_0, \end{aligned} \tag{1.1}$$

where $\mathbf{L}$ is a linear operator in a Banach space $\mathcal{X}$, and $\mathbf{f} \in L^1_{\text{loc}}([0,\infty);\mathcal{X})$, a resolvent kernel is an operator-valued function $\mathbf{S}(t)$ such that the solution of (1.1) is given by

$$\mathbf{x}(t) = \mathbf{S}(t)\mathbf{x}_0 + \int_0^t \mathbf{S}(t-\tau)\,\mathbf{f}\,(\tau)\,d\tau. \tag{1.2}$$

In what follows, $a(t)$ is a real-valued kernel satisfying

$$\begin{aligned} &a \in L^1_{\text{loc}}(\mathbf{R}^+) \text{ is positive, nonincreasing} \\ &\text{and convex on } (0,\infty) \text{ with } 0 < a(0+) \le \infty. \end{aligned} \tag{1.3}$$

Here $\mathbf{R}^+ = [0,\infty)$. The unknown $\mathbf{x}$ takes its values in a Banach space $(\mathcal{X}, ||\cdot||)$ and

$$\begin{aligned} &\mathbf{L} \text{ is a closed linear operator in } \mathcal{X}, \text{ defined on the dense} \\ &\text{domain } \mathcal{D}, \text{ and } \mathbf{L} \text{ is invertible with } \mathbf{L}^{-1} \text{ compact on } \mathcal{X}. \end{aligned} \tag{1.4}$$

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