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SIMPLE QUADRATURE FOR SINGULAR INTEGRALS

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ABSTRACT. In this note quadrature formulae for singular integrals are derived that retain the nice features of Gauss-Chebyshev quadrature, for example the easy to calculate weights and nodes. The reason for these new formulae lies in their application to a special method for solving singular integral equations, where such properties need to be preserved. The study shows that by subtracting the endpoint singularities, formulae converging to the desired integral are obtained. The rates of convergence are shown to depend on the exponents of the Gauss-Jacobi weight function. In practice, fast convergence is attained, giving full accuracy with a very small number of nodes, with execution times comparable to those of Gaussian quadrature.

1. Introduction. Quadrature formulae for integrals possessing a Cauchy principal value singularity have been investigated in the recent literature; and generally, Gaussian quadrature of some form has been used. In [**10**], for example, Gauss-Jacobi formulae have been derived for applications to singular integral equations. The problem of convergence of these rules has been resolved in [**8**]. A different approach has been considered in [**4**], where quadrature formulae have been derived so that the nodes coincide with the "practical" abscissae. Gauss type formulae, however, are not the only way of dealing with the problem [**14**].

The aim of this note is to derive formulae and convergence results for singular integrals with an approach similar to [**4**] where the nodes are prescribed to be "practical" abscissae. Here we also retain the simple weights of the classical Chebyshev quadrature.

The need for such formulae arises in applications to singular integral equations (SIE's) [**18**], where the integral is discretized using Gauss-Chebyshev quadrature, at the expense of accuracy, since the endpoint behavior of the solution is ignored. In this way it is possible to make

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