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A FREDHOLM EQUATION FOR THE HANKEL SINGULAR VALUES OF SYSTEMS WITH DISTRIBUTED INPUT DELAYS

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ABSTRACT. We study a system with continuous input delays (no state delay). We show that its Hankel operator is compact and that the singular values are the square roots of the positive eigenvalues of a Fredholm integral operator.

1. Introduction. Hankel norm and Hankel singular values of distributed systems have been studied by several authors in recent times [**2**]. Some of these papers deal specifically with delay systems ([**3, 1, 10, 11**] for example) usually assuming *discrete* delays. Very few authors considered specifically systems with distributed delays (see [**11**] where a transfer function of a system with distributed delays—and no pole—was studied). Actually, a system with only distributed delays admits a state space representation which is quite simple, and this suggests that we consider this abstract state space representation—described below—as a starting point for formulas which can lead to an alternative characterization of singular values. A consequence of this fact is that the operator $\mathcal{PQ}$ introduced below is compact so that $\sigma(\mathcal{PQ})/\{0\} = \sigma_p(\mathcal{PQ})/\{0\}$ and the singular values can only accumulate at zero.

In this paper we study the control system

$$(1) \qquad \dot{x} = Ax + \int_{-\tau}^{0} B(s)u(t+s)\,ds + B_0u(t) \qquad y = Cx(t).$$

The letters x, y, u denote respectively n-, p- and m-vectors and the matrices have suitable dimensions. The entries of the matrix $B(\cdot)$ are square integrable functions while the remaining matrices are constant.

We assume that A is a stable matrix, i.e., that $\sigma(A) \subseteq \{z, \operatorname{Re} z < 0\}$ so that it is possible to define the Hankel operator $\Gamma : L^2(0, +\infty) \to$

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