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ON USING A MODIFIED NYSTRÖM METHOD TO SOLVE THE 2-D POTENTIAL PROBLEM

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ABSTRACT. In this paper the single-layer potential representation is used to solve the 2-D potential problem with either Dirichlet, Neumann, or Robin boundary conditions. We show that the discrete delta-trigonometric method introduced by Cheng and Arnold [**14**] and the discrete Galerkin method introduced by Atkinson [**9**] obtain the same discrete density. Then we introduce another equivalent method called the modified Nyström method, and show that this method requires only $O(n^2)$ simple operations (instead of $O(n^2 \log n)$) to form the matrices. We also discuss previous convergence results for this method and extend the exponential convergence results to the Robin problem. Finally, we present numerical experiments in order to confirm our theory.

1. Introduction. The two-dimensional potential problem is

$$\Delta U = 0 \quad \text{on } \mathbf{R}^2 \backslash \Gamma, \qquad aU + b\frac{dU}{d\nu} = G \quad \text{on } \Gamma, \tag{1.1}$$

where a and b are constants, U is bounded at infinity, ν is the outward normal, G is analytic and Γ is a simple closed analytic curve. For any harmonic U, there exists a unique Φ satisfying the single-layer potential representation,

$$U(z) = \int_\Gamma \Phi(y) \log|z-y| \, d\Gamma_y \quad \text{for } z \in \mathbf{R}^2, \tag{1.2}$$

if the conformal radius of Γ does not equal 1, e.g., [**9, 13, 14, 49, 50**].

Remark. There are two ways to handle the uniqueness problem when the conformal radius equals 1 as in [**15**]. One approach is to add an

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